

**應用數學系碩士班**

國立臺南大學105學年度 　 　　　　　　　　 招生考試 線性代數 試題卷

1. (a) (10%) 解釋對應矩陣row operations的三種elementary matrices。

(b) (10%) 求出這三種elementary matrices的determinants。

1. (15%) 用矩陣*A*來解釋algebraic multiplicities of an eigenvalue and geometric multiplicities of an eigenvalue；其中the characteristic polynomial of *A* is 。

 

1. (a) (5%) 說明何為一個矩陣的nullity。

(b) (10%) 求出下列矩陣的nullity。



1. (10%) Let $V ∶=\left\{a\_{i}\in R for all 0 \leq i\leq 2\right\}$ with the following operations:

$$\left(a\_{0}+a\_{1} x+a\_{2}x^{2}\right)\vec{+} \left(b\_{0}+b\_{1}x+b\_{2}x^{2}\right)∶= a\_{0} + b\_{0}+\left(a\_{1}b\_{1}\right)x+\left(a\_{2}- b\_{2}\right)x^{2} $$

$$r ∙\left(a\_{0}+a\_{1}x+a\_{2}x^{2}\right)=a\_{0}+ra\_{1}x+\left( r+a\_{2}\right)x^{2}.$$

Show that V together with operations $(\vec{+}, ∙ )$ is not a real vector space. You must explain two rules it violates.

1. (10%) Let

$$W ∶=\left\{a+e+c=0 and d+e=0 \right\}.$$

Show that W is a vector subspace of $M\_{2×3}(R)$ and find the dimension of W.

1. (15%) Define a map $T :R^{4} \rightarrow R^{3}$ by

$T\left(x,y, z,w\right)^{t}=\left(x-2z, y+w, 2x+z \right)^{t}$.

Show that T is a linear transformation. What is the nullity of T? Find a basis for the range space R(T).

1. (15%) Let $h:P\_{2}\left(R\right)\rightarrow R^{2}$ be a linear transformation defined by the matrix H with respect to the bases B and D:

$H= \left( \begin{matrix}1&-2&1\\1&1&-2\end{matrix}\right), B=\left\{x, 1-x, 1+x^{2}\right\}, D= \left\{\left(1,0\right)^{t}, \left(-1,1\right)^{t} \right\}.$

What is $ h( x )$? What is $h( a x^{2}+b x+c)$?