

**資訊工程學系碩士班**

國立臺南大學105學年度 　 　　　　　　　　 招生考試 離散數學與線性代數 試題卷

I. 離散數學 (50%)

1. How many number of arrangements of the letters in “DATABASES” ? (5%)

2. For the polynomial *P*(*x*, *y*, *z*) = (2*x* - 4*y* + 3*z*)10.

 (a) Find the coefficient of *x*2*y*4*z*4 in expansion of *P*(*x*, *y*, *z*). (5%)

 (b) How many distinct terms are there in the complete expansion of *P*(*x*, *y*, *z*)? (5%)

 (c) What is the sum of all coefficients in the complete expansion of *P*(*x*, *y*, *z*)? (5%)

3. Let *k* is a positive integer and *k* ≥ 2, show that if any *k*+1 integers are selected from the set *S* = {1, 2, 3, 4, . . . , 2*k* -1}, there are at least two whose sum is 2*k*. (10%)

4. Let *A*, *B* be two distinct sets, |*A*| = 10, |*B*| = 5. For the functions defined as

 *f*: *A* → *B*, answer the following questions.

 (a) How many functions *f* are defined? (2%)

 (b) How many functions *f* are one-to-one? (2%)

 (c) How many functions *f* are onto? (2%)

5. Let *p*(*x*), *q*(*x*), and *r*(*x*) be the following open statements.

 *p*(*x*): *x*2 - 8*x* + 15 = 0

 *q*(*x*): *x*2 - 2*x* - 3 = 0

 *r*(*x*): *x* > 0

Determine the truth or falsity of the following statements, where the universe is all integers. If a statement is false, provide a counterexample or explanation.

 (a) ∀ *x* [*p*(*x*) → (*q*(*x*) ∧ *r*(*x*))] (2%)

 (b) ∃ *x* [(*q*(*x*) ∧ ¬*r*(*x*)) → *p*(*x*)] (2%)

6. Given the following relations *R* on the sets *S*

　　　(A) *S* = **R**, *x* *R* *y*  ↔ *x* = *y*

　　　(B) *S* = {1, 2, 3, 4, 6, 8, 12, 24}, *x* *R* *y* ↔ *x* | *y*  (**Note:** *x | y* means *x divides y* )

　　　(C) *S* = **Z**, *x* *R* *y* ↔ |*x* – *y*| is odd

Answer the following questions:

 (a) For the relations (A), (B), (C), identify if they have reflexive, symmetric, anti-symmetric, transitive properties. (6%)

 (b) Which relation is an equivalence relation? (2%)

 (c) Which relation is a partial ordering relation? (2%)

II. 線性代數 (50%)

1. Find bases for the subspaces of $R^{4}$ spanned by the following vectors: (10%)

 $(1, 2, 3, 4)$, $(0, -1, 2, 3)$, $(2, 3, 8, 11)$, $(2, 3, 6, 8)$

1. Determine nonzero vectors that are orthogonal to the following vectors. (12%)
	1. $(5, 1, -1)$ (b) $(6, -1, 2, 3)$
2. Use determinants to decide whether the linear transformations defined by the following matrices are one-to-one or not. Give the reason for your answer. (14%)
	1. $A=\left[\begin{matrix}1&2&3\\2&4&6\\0&1&4\end{matrix}\right]$ (b) $B=\left[\begin{matrix}\begin{matrix}2\\\begin{matrix}4\\\begin{matrix}-1\\0\end{matrix}\end{matrix}\end{matrix}&\begin{matrix}\begin{matrix}1\\\begin{matrix}3\\\begin{matrix}7\\-2\end{matrix}\end{matrix}\end{matrix}&\begin{matrix}\begin{matrix}3\\\begin{matrix}0\\\begin{matrix}8\\4\end{matrix}\end{matrix}\end{matrix}&\begin{matrix}0\\\begin{matrix}2\\\begin{matrix}1\\0\end{matrix}\end{matrix}\end{matrix}\end{matrix}\end{matrix}\end{matrix}\right]$
3. Let $A=\left[\begin{matrix}0&2&0\\2&0&0\\0&0&1\end{matrix}\right]$. (14%)
	1. Determine whether $A$ is diagonalizable and, if so, find an invertible matrix $P$ and a diagonal matrix $D$ such that $P^{-1}AP=D$
	2. Compute $A^{8}$