

**應用數學系碩士班**

國立臺南大學106學年度 　 　　　　　　　　 招生考試 微積分 試題卷

1. Find the integral  (8%)
2. Determine whether the integral  converges. (8%)
3. Find the limit . (10%)
4. Find the equation for the tangent line to the curve  at the point. (10%)
5. Let

 (14%)

(1) Prove that  is continuous at .

(2) Find the derivative of .

1. Test the following series for convergence or divergence: (15%)

(1) $\sum\_{n=1}^{\infty }\frac{1}{100+4^{n}}$ .

(2) $\sum\_{n=1}^{\infty }\frac{1}{n^{3/5}}$ .

(3) $\sum\_{n=1}^{\infty }(-1)^{n}\frac{n^{3}}{3^{n}}$ .

1. Define . (20%)
2. Please explain if $\frac{∂f}{∂x}(0,0)$ and $\frac{∂f}{∂y}\left(0,0\right)$ exist.
3. Find the gradient $∇f(0,0)$.
4. Find the maximum rate of change of $f$ at the point $(0,0)$. Also, find the direction in which it occurs (you can describe it in terms of a vector).
5. Suppose that *D* is the half-annulus given by

$$1\leq x^{2}+y^{2}\leq 4, y\geq 0.$$

Evaluate (15%)

$$∬\_{D}^{}sin⁡(x^{2}+y^{2})dA.$$